Performance Analysis of Optical Transmission System with Polarization-Mode Dispersion and Forward Error Correction

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***Abstract*— This letter shows that sufficient interleaving is essential for forward error correction to provide average bit-error-rate (BER) improvement in a high polarization-mode-dispersion (PMD) system. While error correction improves the outrage probability, in term of average BER, error correction cannot extend the tolerable PMD level and provides no performance improvement when PMD is larger than 0.2-bit interval.**

***Index Terms*— Error correction, fiber impairments, polarization-mode dispersion.**

I. Introduction

IN HIGH-CAPACITY long-distance optical transmission systems, polarization-mode dispersion (PMD) [1]–[3] in single-mode fiber imposes a potential limit in the maximum bit-rate-distance product of the system [4]–[7]. PMD could contribute to bit-error-rate (BER) deterioration, performance variations or system fading even in the moderate bit-rate-distance-product regime. Forward-error correction (FEC) coding was found to be an effective method to improve the system performance of optical transmission systems [8]–[15]. FEC is used in trans-oceanic transmission systems [10]–[11], or to increase the repeaterless transmission distance [8], [12]–[13]. FEC is expected to improve the system performance deteriorated by PMD [11] and to increase the tolerable PMD level accordingly.

The performance of optical transmission systems with high-PMD and FEC is analyzed. It is found that while FEC provides average BER and outrage probability improvement at most PMD levels, to some extent and in contrast to conventional belief, without sufficient interleaving, FEC does not increase the tolerable PMD level for the same average BER level. However, the authors know no practical method to provide the function of interleaving. Assumed that PMD is a fixed static process [16] instead of a random process [1]–[7], previous analysis [16] had already found that FEC cannot improve the system performance of high PMD systems. The analysis of this letter is more accurate and also provides detail results.

The remaining part of this letter is organized as follow: Section II analyzes the system with PMD and FEC; Section III presents numerical results on average BER and outrage probability; Section IV provides some discussion; and Section V gives the conclusion.

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II. Performance Analysis

PMD arises in single-mode fibers when a perfect circular symmetry in geometry and stress within the fiber structure is disrupted during the process of fiber manufacturing, cabling or installation. PMD can be characterized by the differential group delay (DGD) between the two signals traveling along each of the two PSP's. It is shown that, if the fiber length is much longer than the correlation length of the disturbance that caused the change of symmetry in fiber geometry, for a fiber with mean DGD value of $\langle\Delta\tau\rangle$, instead of a fixed DGD [16], the DGD $\Delta\tau$ between the two PSP's follows a Maxwellian probability density function [2]–[3], [5]–[7]:

$$P_{\langle\Delta\tau\rangle}(\Delta\tau) = \frac{32\cdot\Delta\tau^2}{\pi^2\langle\Delta\tau\rangle^3}\,\exp\left(-\frac{4\cdot\Delta\tau^2}{\pi\langle\Delta\tau\rangle^2}\right)\qquad 0<\Delta\tau<\infty. \tag{1}$$

If an optical transmission system has a pulse shape of $g(t)$, with a DGD of $\Delta\tau$, with the worst-case assumption that the two PSP's share equal optical power, the pulse is broadened to

$$g_{\Delta\tau}(t) = (g(t-\Delta\tau/2) + g(t+\Delta\tau/2))/2. \tag{2}$$

The broadening of the optical pulse deteriorates the system by inducing intersymbol interference. The BER performance with PMD has been evaluated for Butterworth filtering [6] and trapezoidal pulses [7]. Without going into detail, we assume that the BER function is $p_e(Q,\Delta\tau)$ where $Q = (I_1 - I_0)/(\sigma_1+\sigma_0)$ is the signal-to-noise ratio (SNR) of the system, I_1, σ_1 and I_0, σ_0 are the signal amplitudes and noise variances at the "1" and "0" levels, respectively. The function $p_e(Q,\Delta\tau)$ depends on the receiver filtering [6], the rise/fall time and the amount of amplifier noise of the system [7].

Without FEC, the overall BER of the system as a function of PMD and system Q-factor can be evaluated according to [7]

$$\mathrm{BER}(\langle\Delta\tau\rangle, Q) = \int_0^\infty p_{\langle\Delta\tau\rangle}(\Delta\tau)p_e(q,\Delta\tau)\,d\Delta\tau. \tag{3}$$

FEC provides a way to improve the system performance. Although FEC can be incorporated using the undefined bits in an STM frame [14]–[15] without data-rate expansion, the Reed–Solomon (RS) code with 7% data-rate expansion in GF(256) and 8-bit correction capability is the most popular FEC for system experiments [8]–[13]. For an (n,k) block code with t error-correction capability and free distance of d, the relationship between the input and output error probability is well approximated by [17],

$$p_{\mathrm{FEC}}(p_i) = \frac{d}{n}\sum_{i=t+1}^{d}\binom{n}{i}p_i^i(1-p_i)^{n-1} + \frac{1}{n}\sum_{i=d+1}^{n} i\binom{n}{i}p_i^i(1-p_i)^{n-i} \quad (4)$$

where p_i is the error probability without FEC. The popular RS(255, 239) code [8]–[13] has a $t = 8$ and $d = 17$.

With ideal interleaving, each bit in the FEC has an independent BER given by (3). The resulting BER with FEC is

$$\mathrm{BER} = p_{\mathrm{FEC}}(\mathrm{BER}(\langle\Delta\tau\rangle, Q)) \text{ with sufficient interleaving.} \quad (5)$$

If all bits within each FEC block experience the same amount of PMD fading, the resulting BER is

$$\mathrm{BER} = \int_0^{\infty} p_{\langle\Delta\tau\rangle}(\Delta\tau)p_{\mathrm{FEC}}(p_e(Q,\Delta\tau))\, d\Delta\tau \quad \text{with insufficient interleaving.} \quad (6)$$

Interleaving is a technique such that adjacent bits in a FEC block are transmitted with some separation bit interval called interleaving depth. For sufficient interleaving with large enough interleaving depth, adjacent bits in a FEC block experience independent error even the channel provided burst errors. FEC is usually implemented at the STM-1 level with an uncoded data rate of 155.52 Mb/s and each FEC block is transmitted within a short time of 1.54 μs. A STM-N system has an interleaving depth of N. In digital communication systems, how fast the channel change is described by the *coherence time*, Δt_c [18] of the channel. Whether the BER calculated by (5) or (6) can be applied depends on $\Delta t_c \ll$ 1.54 or $\Delta t_c \gg$ 1.54 μs, respectively. From measurements presented in [1], depending on the ambient environment, the BER faded within minutes or hour, which is considerable larger than 1.54 μs. We may conclude that the BER provided by (6) may be a more suitable one to be used for high-speed optical transmission systems with only electrical interleaving. Furthermore, the BER provided by (6) may be a worst-case or conservative estimation of the system performance.

III. Numerical Results

With the worst-case assumption that the two PSP's have equal optical power, Fig. 1 shows the required receiver Q-factor to achieve a BER of 10^{-12} as a function of PMD with and without FEC. The PMD is normalized with the bit-interval of the uncoded bit-stream, T_b. Since the BER depends on rise/fall time [7] and/or the low-pass filter in the receiver [5], we assume that the received waveform is rectangular pulses with a 0.75-bit rate fourth-order Bessel–Thompson receiver filter [19] or a trapezoidal pulse shape [7] with a rise/fall time of $0.5T_b$. We also assume that the noise variances at both the "0" and "1" level are identical, i.e., $\sigma_1 = \sigma_0$, for simplicity. Although FEC provides a 7-dB improvement in receiver Q-factor in low PMD, FEC with insufficient interleaving provides no performance improvement when PMD is larger than $0.2T_b$. However, with sufficient interleaving such that each bit has independent DGD, a large performance improvement is provided until the PMD increases to about $0.4T_b$. A high PMD system with FEC and insufficient interleaving performs worse than a system without FEC because of the 7% increase in both the bit rate and the PMD relative to the bit interval.

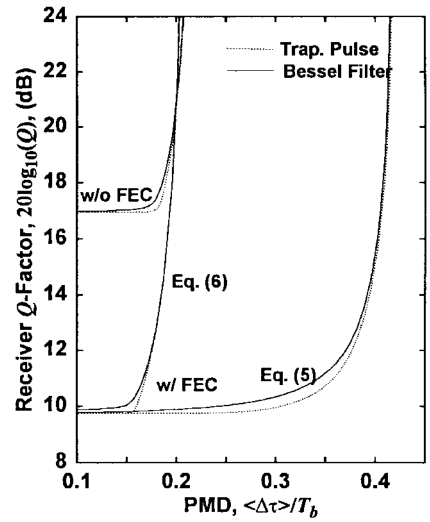

Fig. 1. Required receiver Q-factor to achieve a BER of 10^{-12} as a function of PMD with and without FEC. BER is evaluated with (5) and (6) for comparison.

With insufficient interleaving, the system performance may be characterized by outrage probability [5]–[6] (for example, the probability that $p_e(Q, \Delta\tau) > 10^{-12}$) instead of the average BER [7]. While both the outrage probability and the average BER are important, average BER has simple physical meaning. Fig. 2 shows the required Q-factor for an outrage probability of 0.1, 10^{-3}, and 10^{-6} with Bessel–Thompson filter as a function of Q-factor. The outrage probabilities larger than an instantaneously BER of 10^{-12} are calculated by $P\{p_{\mathrm{FEC}}(p_e(Q, \Delta\tau)) > 10^{-12}\}$ and $P\{p_e(Q, \Delta\tau) > 10^{-12}\}$ for system with and without FEC, respectively. The required Q-factor is decreased by using FEC in most PMD level and the tolerable PMD level also increases. At a Q-factor of 24 dB, the PMD level increases about 10% using FEC.

IV. Discussion

Fig. 1 indicates that FEC provides very little improvement for systems without sufficient interleaving but Fig. 2 shows some improvement in outrage probability. We would like to provide a qualitative explanation of this effect.

The performance of FEC can be described by the input/output error probability relationship (4). FEC provides a large BER improvement when the input BER is low, but almost no improvement when the input BER is high. For example, for an input BER of 10^{-3}, the RS(255, 239) code provides an output BER around 10^{-12}. For an input BER of 10^{-2}, the code provides an output BER about 10^{-4}. For an input BER larger than 2×10^{-12}, there is no improvement in the output BER because the number of bit errors in each FEC block is likely to be larger than t = 8. Comparing the BER

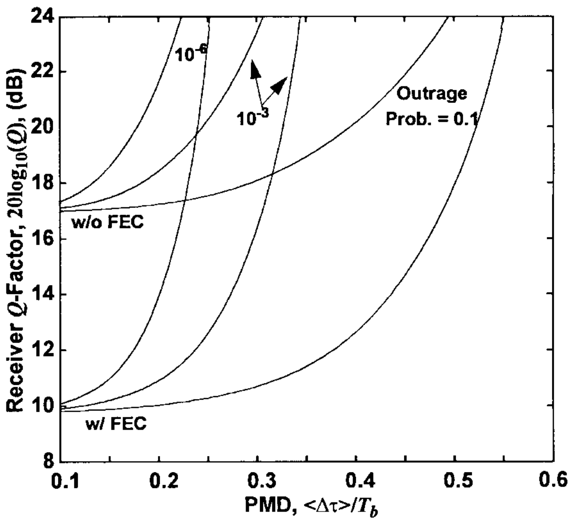

Fig. 2. Required receiver Q-factor to achieve outrage probability of 0.1, 10^{-3}, and 10^{-6} as a function of PMD with and without FEC.

equations (3) and (6) with and without FEC, the BER may be dominated by the part of integration in which both the DGD $\Delta\tau$ and also $p_e(Q, \Delta\tau)$ are large. However, when $p_e(Q, \Delta\tau)$ is large, $p_e(Q, \Delta\tau)$ itself in (3) and $p_{\rm FEC}(p_e(Q, \Delta\tau))$ in (6) have a very small difference such that the resulting BER has little difference.

Outrage probability, without the integration in (3) and (6), is directly equal to the probability of $p_e(Q, \Delta\tau)$ and $p_{\rm FEC}(Q, \Delta\tau))$ larger than a fixed instantaneously BER. Usually, the specific BER is small (for example, 10^{-12} in Fig. 2), an improvement in outrage probability can be observed. If the specific BER is larger than 10^{-4}, we may expect small improvement in outrage probability.

As indicated earlier, each FEC block transmits within 1.54 μs if FEC is implemented in STM-1 level. Providing sufficient interleaving by buffering the data may be difficult because the coherence time of PMD fading lasts for minutes or even longer [1]. The authors know no practical method to provide the function of interleaving by randomizing the DGD of adjacent bits of a FEC block. Although to be able to improve the system performance by randomizing the coupling ratio between two PSP's, polarization scrambling will not change the DGD and cannot function as an interleaver.

Theoretically, the input/output BER formula (4) provides a very good approximation for block code [17]. However, the experimental input/output error probability relationship usually performs not as well as the theoretical prediction [8], [10]. In practice, Figs. 1 and 2 may over-estimate the system improvement with FEC, but it may provide the best achievable system improvement if FEC functions as predicted.

V. Conclusion

The effect of FEC in high PMD system is analyzed in this paper. This paper indicates that sufficient interleaving is essential for FEC to provide an average BER performance improvement. Although able to improve the outrage probability, FEC with insufficient interleaving cannot extend the tolerable PMD level for a fixed average BER and almost no improvement may be provided by FEC when PMD is larger than 0.2-bit interval. While error correction with sufficient interleaving provides a large performance improvement at all PMD levels and the tolerable PMD level is increased to about 0.4-bit interval, there is no known practical method to provide the function of interleaving.

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